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Direct URCA process in neutron stars with strong magnetic fields

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Abstract: We calculate the emissivity for the direct URCA process in strongly magnetized, degenerate matter in neutron stars, under β-equilibrium. We show that, if the magnetic field is large enough for protons and electrons to be confined to their ground Landau levels, the field-free threshold condition on proton concentration no longer holds, and direct URCA reactions are open for an arbitrary proton concentration. Direct URCA process leads to an early phase of fast neutron star cooling. This circumstance allows us to constrain the initial magnetic field inside observed pulsars.

Keywords: Dark Matter, High Energy Cosmic Rays, Physics of the Early Universe.

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Contents

1. Introduction

It is now well established that pulsars are neutron stars with a very strong magnetic field. The surface magnetic field manifests itself by synchrotron radiation from the pulsar magnetosphere and its magnitude for young neutron stars has been estimated [1] to be as large as $10^{12} - 10^{14}$ G. The internal magnetic field may not manifest at the surface and therefore its strength is controversial but, because of the high conductivity of the core, the value of the magnetic field is expected to be further increased . Considering the flux throughout the star as a simply trapped primordial flux and taking into account a possible twisting of the field, an effect that would intensify the field additionally by several orders of magnitude, one obtains an internal magnetic field intensity of the order 10^{18} G. Such a simple hypothesis meets, however, a reasonable criticism, because nascent neutron stars are convective. Some works have been devoted to develop an alternative to the origin of the internal magnetic field [2], based on the convective dynamo mechanism, which gives rise to a magnetic field of the order $10^{14} - 10^{15}$ G inside neutron stars. However, in spite of the fact that we do not know so far any appropriate mechanism to produce more intense magnetic fields, the scalar virial theorem [3] allows the field magnitude to be as large as 10^{18} G. The time duration of such a strong large-scale magnetic field decay via the mechanism of ambipolar diffusion might be about a few decades or a hundred year [4]. Magnetic fields of this magnitude, and even more intense, have been recently discussed [5, 6, 7, 8] as possibly existing inside some neutron stars and other compact astrophysical objects. A numerical study [9] shows that such an enormous magnetic field causes a dramatic deformation of the neutron star. The magnetic pressure, in this case, exceeds the fluid pressure everywhere on the symmetry axis and the neutron star configuration deviates from the spherical configuration showing a pinch along the symmetry axis. Nevertheless, the above configuration is stable up to a magnetic field $B \leqslant 10^{18}$ G.

To constrain the magnetic field inside neutron stars, it would be helpful to search for any observable consequences of a superstrong magnetic field. For example, one can observe a rapid motion of the neutron star under the recoil action, due to anisotropic neutrino emission induced by an intense magnetic field [10, 11, 12, 13] or one can analyze the effect of the magnetic field on the neutron star mass [8].

Moreover, if superstrong magnetic fields exist in the interior of young neutron stars, new processes would be open, and standard processes would also be modified by the magnetic field, thus changing the evolution of those stars. Studying the influence of these modifications and confrontation with observational data can be used to put some bounds on the initial magnetic field. The aim of this paper is to consider one of these modifications, and to motivate further research on this field.

As we will show, a crucial mechanism when considering the cooling of neutron stars with superstrong magnetic fields are the direct URCA reactions

$$p + e^- \xrightarrow{B} n + \nu \,, \tag{1.1}$$

$$n \xrightarrow{B} p + e^- + \overline{\nu} \,, \tag{1.2}$$

where the symbol B on the top of the arrow means that the reactions rates have to be calculated for particle states in the magnetic field.

We will consider degenerate, nonrelativistic nuclear matter and ultrarelativistic degenerate electrons under β-equilibrium. Matter is assumed to be totally transparent for neutrinos and antineutrinos. We also assume that at zero temperature the neutron star is β-stable, but at non-zero temperature $T \ll \varepsilon_i^F$ $(i = n, p, e)$ reactions (1.1) and (1.2) proceed near the Fermi energies ε_i^F of the participating particles. In the field-free case, for reactions $p + e^- \to n + \nu$; $n \to p + e^- + \overline{\nu}$ to take place, momentum conservation near the Fermi surfaces requires the following inequality among the Fermi momenta of the proton (p_F), the electron (k_F) and neutron (q_F):

$$p_F + k_F \geq q_F \,. \tag{1.3}$$

Together with the charge neutrality condition, the above inequality leads to the threshold for proton concentration $Y_p \equiv n_p/\left(n_p + n_n\right) \geq 1/9$, where n_p and n_n are the number densities of protons and neutrons, respectively. This means that, in the field-free case, direct URCA reactions are strongly suppressed by Pauli blocking in the neutron-rich nuclear matter (assuming matter is formed only by protons, neutrons and electrons).

In magnetized media, transversal momenta of charged particles are defined only within an accuracy $\Delta p_\perp \sim \sqrt{eB}$. We show in the following that strong magnetic fields $\sqrt{eB} \sim q_F$ break down condition eq. (1.3) so that direct URCA processes are allowed in the degenerate n-p-e system under β-equilibrium for an arbitrary proton concentration. We examine reactions (1.1) and (1.2) in a constant magnetic field large enough so that[1]

[1]Here e is the absolute value of electron charge. For electrons having a Fermi momentum of 50 Mev one obtains $B > 2 \times 10^{17}$ G. We set $\hbar = c = k_B = 1$.

$B > k_F^2/(2e)$. In this case the Fermi energy of degenerate electrons is smaller than the energy of the first excited Landau level $\sqrt{m^2+2eB}$. In other words, at zero temperature, all electrons occupy only the ground Landau band and are polarized in the direction opposite to the magnetic field . The number density of electrons reads

$$n_e = \frac{eB}{4\pi^2}\int_{-k_F}^{k_F} dk_3 = \frac{eB}{2\pi^2}k_F\,, \tag{1.4}$$

where k_F is the Fermi momentum of degenerate electrons. Following charge neutrality $n_p = n_e$, one obtains the proton Fermi momentum p_F to be equal to k_F. Therefore, if $B > k_F^2/(2e)$, then degenerate protons are also confined to the ground Landau state and are totally polarized in the direction parallel to the external magnetic field. We choose our coordinate system in such a way that $\mathbf{B} = (0,0,B)$. As will be obtained through the next sections, under the conditions mentioned above the direct URCA process (1.1) and (1.2) gives raise to a large neutrino luminosity during the first 10 yr. of the strongly magnetized neutron star. Such large luminosity would lead to an anomalously cold neutron star, a fact that can be confronted with observational data and eventually used to put some bounds on the initial magnetic field.

2. Calculation of the emissivity

The emissivity for reactions (1.1) and (1.2) can be computed using standard charged-current β-decay theory. The matrix element for the V-A interaction reads:

$$M_\sigma = \frac{G}{\sqrt{2}}\int d^3r\,\overline{\Psi}_{n\sigma}(\mathbf{r})\,\gamma_\mu\,(1-g_A\gamma_5)\,\Psi_p(\mathbf{r})\,\overline{\Psi}_\nu(\mathbf{r})\,\gamma^\mu\,(1-\gamma_5)\,\Psi_e(\mathbf{r})\ , \tag{2.1}$$

where $\sigma = +$ $(\sigma = -)$ corresponds to neutrons polarized along (in the opposite direction to) the magnetic field, $\Psi_\nu(\mathbf{r})$ and Ψ_e represent the neutrino and electron fields , respectively, while Ψ_p and Ψ_n stand for the proton and neutron; $G = G_F\cos\theta_C$ with θ_C being the Cabibbo angle, and $g_A \simeq 1.261$ is the axial-vector coupling constant. By using the asymmetric (Landau) gauge $\mathbf{A}=(0,Bx,0)$, the four-dimensional polarized wave functions can be expressed in terms of stationary states in the normalization volume $V = L_1L_2L_3$. The electron wave function, corresponding to the ground Landau band with energy $\varepsilon_0 = \sqrt{m^2+k_3^2}$ reads

$$\Psi_e(\mathbf{r}) = \frac{1}{\sqrt{2\varepsilon_0}\sqrt{L_2L_3}}\exp(-i\varepsilon_0 t)\exp\left[i\left(k_3z+k_2y\right)\right]\varphi_0(\xi)u_e\,. \tag{2.2}$$

The function $\varphi_0(\xi)$ is the eigenfunction of the one-dimensional harmonic oscillator, normalized with respect to $\xi = \sqrt{eB}\,(x+k_2/(eB))$

$$\varphi_0(\xi) = \left(\frac{eB}{\pi}\right)^{1/4}\exp(-eB\xi^2/2) \tag{2.3}$$

and

$$u_e = \frac{1}{\sqrt{\varepsilon_0 + m}} \begin{pmatrix} 0 \\ \varepsilon_0 + m \\ 0 \\ -k_3 \end{pmatrix} \tag{2.4}$$

with k_2, k_3 the electron momenta in the y and z directions, respectively. In order to take into account strong interactions in nuclear matter, we consider nonrelativistic protons, with an effective mass M_p^*, moving in the self-consistent uniform nuclear potential U_p. Then the proton wave function reads

$$\Psi_p(\mathbf{r}) = \frac{1}{\sqrt{L_2 L_3}} \exp(-iE_0 t) \exp\left[i\left(p_3 z + p_2 y\right)\right] \varphi_0(\zeta) \begin{pmatrix} W_+ \\ 0 \end{pmatrix}, \tag{2.5}$$

where p_2, p_3 are proton momenta in the y and z directions, respectively, and $\zeta = \sqrt{eB}\,(x - p_2/(eB))$. Polarized protons are given by the non-relativistic spin-up bispinor W_+ normalized by the condition $W_+^\dagger W_+ = 1$. By introducing the anomalous magnetic moment of the proton, its energy reads [14] $E_0 \simeq \widetilde{M}_p + \varepsilon_p + U_p$ where $\varepsilon_p = p_3^2/\left(2\widetilde{M}_p\right)$ and

$$\widetilde{M}_p = M_p^* - \frac{e}{2M_p^*}\left(g_p - 1\right) B \tag{2.6}$$

with the proton's Lande factor $g_p = 2.79$. In a similar way, we consider non-relativistic neutrons, of effective mass M_n^*, moving in the self-consistent uniform nuclear potential U_n. One can describe neutron polarization by non-relativistic bispinors W_σ ($\sigma = \pm$). They are normalized by the condition $W_\sigma^\dagger W_\sigma = 1$. Then the neutron wave function reads

$$\Psi_{n\sigma}(\mathbf{r}) = \frac{1}{\sqrt{L_2 L_3 L_3}} \exp(-iE_{q\sigma} t) \exp\left[i\left(q_3 z + q_2 y + q_1 x\right)\right] \begin{pmatrix} W_\sigma \\ 0 \end{pmatrix}. \tag{2.7}$$

Here the energy of a neutron with spin polarization σ is $E_{q\sigma} = M_n^* + \varepsilon_{q\sigma} + U_n$ with

$$\varepsilon_{q\sigma} = \frac{q^2}{2M_n^*} - \sigma \mu_n B, \tag{2.8}$$

which incorporates the neutron interaction with the external magnetic field due to its anomalous magnetic moment $\mu_n = g_n e/(2M_p)$ with $g_n = -1.91$. In accordance to eq.(2.8), neutrons with different polarizations $\sigma = \pm$ have different momenta $q_{F\pm}$ at the Fermi surface $\varepsilon_{q\pm} = \varepsilon_n^F$. Taking into account

$$\varepsilon_n^F = \frac{q_{F\pm}^2}{2M_n^*} \pm \mu_n B, \tag{2.9}$$

one finds $q_{F-}^2 = q_{F+}^2 + 4M_n^* \mu_n B$. Explicit evaluation of the matrix elements M_σ in (2.1) yields

$$\left.\begin{matrix} |M_+|^2 \\ |M_-|^2 \end{matrix}\right\} = \left.\begin{matrix} 2\left(1 + g_A^2\right)\left(\omega + \kappa_3\right) \\ 8 g_A^2 \left(\omega - \kappa_3\right) \end{matrix}\right\} G^2 \left(\varepsilon_0 + k_3\right) \exp\left(-\frac{1}{2} \frac{q_1^2 + \left(p_2 + k_2\right)^2}{eB}\right). \tag{2.10}$$

The neutrino momentum is $\kappa = (\kappa_1, \kappa_2, \kappa_3)$ and its energy is $\omega = |\kappa|$. We have neglected the neutrino momentum $\kappa = \omega \sim T$, when compared to neutron momenta $q_{F\pm}$. The neutrino emissivity reads

$$\epsilon_\nu = \frac{1}{V^2 L_2^2 L_3^2} \int_{-eBL_1/2}^{eBL_1/2} \frac{L_2 dk_2}{2\pi} \int \frac{L_3 dk_3}{2\pi} \int_{-eBL_1/2}^{eBL_1/2} \frac{L_2 dp_2}{2\pi} \int \frac{L_3 dp_3}{2\pi} \int \frac{V d^3 q}{(2\pi)^3} \int \frac{V d^3 \kappa}{(2\pi)^3} \times$$
$$\times \frac{\omega}{2\omega 2\varepsilon_0} 2\pi \left[\begin{array}{c} |M_+|^2 \delta\left(E_{q+} + \omega - E_0 - \varepsilon_0\right)\left(1 - f_{n+}\right) f_e f_p \\ + |M_-|^2 \delta\left(E_{q-} + \omega - E_0 - \varepsilon_0\right)\left(1 - f_{n-}\right) f_e f_p \end{array} \right] \times$$
$$\times 2\pi\delta\left(q_3 + \kappa_3 - p_3 - k_3\right) 2\pi\delta\left(q_2 + \kappa_2 - p_2 - k_2\right) . \tag{2.11}$$

Here $(1 - f_{n\sigma}) f_e f_p$ are statistical factors corresponding to Fermi-Dirac distributions of particles with chemical potentials $\psi_n \simeq M_n^* + U_n + \varepsilon_n^F$, $\psi_e \simeq \varepsilon_e^F$ and $\psi_p \simeq \widetilde{M}_p + \varepsilon_p^F + U_p$. The Fermi energy of neutrons is defined in (2.9). For electrons and protons, they are given by

$$\varepsilon_e^F = \sqrt{m^2 + k_F^2}\,, \qquad\qquad \varepsilon_p^F = \frac{p_F^2}{2\widetilde{M}_p} . \tag{2.12}$$

With these definitions one has

$$f_{n\pm} = \frac{1}{\exp\left[\left(\varepsilon_{q\pm} - \varepsilon_n^F\right)/T\right] + 1} , \tag{2.13}$$

$$f_e = \frac{1}{\exp\left[\left(\varepsilon_0 - \varepsilon_e^F\right)/T\right] + 1} , \tag{2.14}$$

$$f_p = \frac{1}{\exp\left[\left(\varepsilon_p - \varepsilon_p^F\right)/T\right] + 1} . \tag{2.15}$$

Since the integrand in eq. (2.11) depends on p_2 and k_2 only through the combination $p_2 + k_2$, we can introduce a new variable $k_2 + p_2 \to p_2$ and perform integration over k_2. We can also neglect the small neutrino momentum in the momentum conservation δ-functions. Thus, we obtain

$$\epsilon_\nu = \frac{G^2 eB}{(2\pi)^7} \int dk_3 dp_2 dp_3 d^3 q d^3 \kappa\, \delta\left(q_3 - p_3 - k_3\right) \delta\left(q_2 - p_2\right) \frac{\left(\varepsilon_0 + k_3\right)}{\varepsilon_0} \exp\left(-\frac{q_1^2 + p_2^2}{2eB}\right) \times$$
$$\times \left[\begin{array}{c} \frac{1}{2}\left(1 + g_A^2\right)\left(\omega + \kappa_3\right) \delta\left(M_n^* - \widetilde{M}_p + U_n - U_p + \varepsilon_{q+} + \omega - \varepsilon_p - \varepsilon_0\right)\left(1 - f_{n+}\right) f_e f_p \\ + 2g_A^2 \left(\omega - \kappa_3\right) \delta\left(M_n^* - \widetilde{M}_p + U_n - U_p + \varepsilon_{q-} + \omega - \varepsilon_p - \varepsilon_0\right)\left(1 - f_{n-}\right) f_e f_p \end{array} \right] . \tag{2.16}$$

The integrals can be performed in a straightforward way by using the following identity:

$$f_e\left(\varepsilon_0\right) f_p\left(\varepsilon_p\right)\left(1 - f_n\left(\varepsilon_{n\pm}\right)\right) \delta\left(M_n^* - \widetilde{M}_p + U_n - U_p + \varepsilon_{q\pm} + \omega - \varepsilon_p - \varepsilon_0\right) =$$
$$= \int_{-\infty}^{\infty} d\varepsilon_3 \int_{-\infty}^{\infty} d\varepsilon_2 \int_{-\infty}^{\infty} d\varepsilon_1 f_e\left(\varepsilon_1\right) f_p\left(\varepsilon_2\right)\left(1 - f_n\left(\varepsilon_3\right)\right) \times$$
$$\times \delta\left(M_n^* - \widetilde{M}_p + U_n - U_p + \varepsilon_3 + \omega - \varepsilon_2 - \varepsilon_1\right) \times$$
$$\times \delta\left(\varepsilon_3 - \varepsilon_{n\pm}\right) \delta\left(\varepsilon_2 - \varepsilon_p\right) \delta\left(\varepsilon_1 - \varepsilon_0\right) . \tag{2.17}$$

Then, due to Pauli blocking, the main contribution to the integrals comes from $\varepsilon_1 \approx \varepsilon_e^F$, $\varepsilon_2 \approx \varepsilon_p^F$ and $\varepsilon_3 \approx \varepsilon_n^F$. By this reason, one can substitute $\delta\left(\varepsilon_n^F - \varepsilon_{n\pm}\right)\delta\left(\varepsilon_p^F - \varepsilon_p\right) \cdot$ $\cdot\, \delta\left(\varepsilon_e^F - \varepsilon_0\right)$ instead of $\delta\left(\varepsilon_3 - \varepsilon_{n\pm}\right)\delta\left(\varepsilon_2 - \varepsilon_p\right)\delta\left(\varepsilon_1 - \varepsilon_0\right)$. Considering electrons as ultrarelativistic, we obtain that $(\varepsilon_0 + k_3)$ is not small only when for $k_3 \geq 0$.

Under the β-equilibrium assumption, one has the following relationship:

$$f_e\left(\varepsilon_0\right) f_p\left(\varepsilon_p\right)\left(1 - f_n\left(\varepsilon_{n\pm}\right)\right) = \left(1 - f_e\left(\varepsilon_0\right)\right)\left(1 - f_p\left(\varepsilon_p\right)\right) f_n\left(\varepsilon_{n\pm}\right) . \tag{2.18}$$

Therefore, the neutron decay process (1.2) gives the same energy loss rate as process (1.1), although giving final antineutrinos. We finally arrive to the following total neutrino plus antineutrino emissivity:

$$\begin{aligned}\epsilon = \epsilon_0 \frac{M_n^*}{p_F}\frac{\widetilde{M}_p}{M_p}\frac{B}{B_0}\Big[&\left(1+g_A^2\right)\Theta\left(q_{F+}^2 - 4p_F^2\right)\exp\left(-\frac{q_{F+}^2 - 4p_F^2}{2eB}\right) + \\ &+4\left(1+g_A^2\right)\Theta\left(q_{F+}^2\right)\exp\left(-\frac{q_{F+}^2}{2eB}\right) + \\ &+2g_A^2\Theta\left(q_{F-}^2 - 4p_F^2\right)\exp\left(-\frac{q_{F-}^2 - 4p_F^2}{2eB}\right) + 8g_A^2\Theta\left(q_{F-}^2\right)\exp\left(-\frac{q_{F-}^2}{2eB}\right)\Big],\end{aligned} \tag{2.19}$$

where the charge neutrality condition $k_F = p_F$ was used and $\epsilon_0 = 2\frac{457\pi}{40320}G^2 M_p^3 T^6 = 1.04\times 10^{27}\, T_9^6\,(\mathrm{ergs/cm^3 s})$, with T_9 the temperature in units of $10^9 K$ and $B_0 = M_p^2/e \simeq 1.5\times 10^{20} G$. Since both electrons and protons are at the ground Landau level, the third-component momentum conservation reads either $q_3 = k_F + p_F$, when proton and electron go in the same direction, along the magnetic field, or $q_3 = k_F - p_F$ when the initial particles move in opposite directions . Since $k_F = p_F$ and $q_{F\pm}^2 > q_3^2$, four possibilities are open : $q_{F+}^2 > (2p_F)^2$, $q_{F-}^2 > (2p_F)^2$, $q_{F+}^2 > 0$ and $q_{F-}^2 > 0$. This is the origin of the four terms in the latter equation, which are multiplied by their corresponding step functions, where $\Theta(x)$ gives $+1$ if the argument exceeds 0, and is 0 otherwise. As we will show later, opening or closing of these channels will give rise to jumps on the emissivity.

3. Results

In order to estimate numerically the emissivity of the direct URCA process in a strongly magnetized neutron star, we consider nuclear matter and electrons in β-equilibrium within a nonrelativistic Hartree approach in the linear σ-ω-ρ model, as it was made (relativistically) in [7], but with taking into account the anomalous magnetic moments of proton and neutron in the energies for these particles. Then the interaction energies for protons and neutrons are given by

$$U_p = \left(\frac{g_\omega}{m_\omega}\right)^2 n_b + \frac{1}{4}\left(\frac{g_\rho}{m_\rho}\right)^2 \rho_3 , \tag{3.1}$$

$$U_n = \left(\frac{g_\omega}{m_\omega}\right)^2 n_b - \frac{1}{4}\left(\frac{g_\rho}{m_\rho}\right)^2 \rho_3 , \tag{3.2}$$

where $n_b = n_p + n_n$ is the total number density of baryons and $\rho_3 = n_p - n_n$. Also,

$$n_p = n_e = \frac{M_p^2}{2\pi^2}\frac{B}{B_0}p_F\,, \tag{3.3}$$

$$n_n = \frac{1}{6\pi^2}\left(q_{F+}^3 + q_{F-}^3\right)\,. \tag{3.4}$$

The parameters for the coupling constants and mesons masses are taken from [15] to be $g_\omega^2\,(M/m_\omega)^2 = 273.87$, and $g_\rho^2\,(M/m_\rho)^2 = 97$. In this model, mass $M = 939MeV$ is the same for proton and neutron.

To make our estimates for the emissivity, we solved for variables q_{F+} and p_F the β-equilibrium condition

$$\psi_p + \psi_e = \psi_n \tag{3.5}$$

with the help of equations (3.1)-(3.4) and $q_{F-}^2 = q_{F+}^2 + 4M_n^*\mu_n B$, for a given baryon density n_b. In order to be fully consistent with incorporating magnetic moments of proton and neutron, one would also like to find the nucleon effective mass M^* from this modified model. However, this would add some (perhaps unessential) calculation problems. Our purpose here is to point out some qualitative new results and motivate more realistic computations. For this reason , we have chosen for our estimates the values of the effective mass that are given by [7]. More precisely, we have taken $M^* = 0.6M$ for n_b equal to nuclear saturation density, which is $n_0 = 0.15fm^{-3}$ in this model, and $M^* = 0.3M$ for $n_b = 2n_0$.

In fig. 1 we have plotted the emissivity of the direct URCA process, calculated from eq. (2.19) as a function of the magnetic field intensity (in units of B/B_0) for two values of the baryon density : $n_b = n_0$ and $n_b = 2n_0$. We have chosen, as a representative value for the temperature, $T = 10^9K$. As one can see from this figure, if the magnetic field is larger than $B \simeq 7 \times 10^{17}G$, the $\nu\overline{\nu}$ emissivity from the neutron star is many orders of magnitude larger than that in the standard cooling scenario. Another feature which is readily observed is that, as the magnetic field increases, jumps on the emissivity can appear. The first jump in the curve $n_b = n_0$ is due to $q_{F-}^2 - 4p_F^2$ becoming negative when $B/B_0 \gtrsim 0.014$. The second, much smaller one, at $B/B_0 \sim 0.026$, corresponds to q_{F-}^2 becoming zero. For the curve $n_b = 2n_0$ we observe a different situation : $q_{F-}^2 - 4p_F^2$ is always negative, but an abrupt increase appears due to the fact that $q_{F+}^2 - 4p_F^2$ is first negative and, as the magnetic field increases, it becomes positive at $B/B_0 \sim 0.03$.

Due to the transitions discussed above, it becomes difficult to find an overall analytic formula to fit the emissivity dependence upon the magnetic field intensity, although it is possible to find a fitting formula which describes the gross tendency of ϵ (without the details described above) for a particular value of the baryon density. We have found, when $n_b = n_0$ that the emissivity can be approximated by

$$\epsilon \simeq 2.40 \times 10^{27} b\,(2.855 - 4.255b)\left(23.05e^{-0.037/b} + 5.763e^{-0.02/b}\right) T_9^6 \mathrm{erg\,cm^{-3}s^{-1}}\,, \tag{3.6}$$

where $b = B/B_0$.

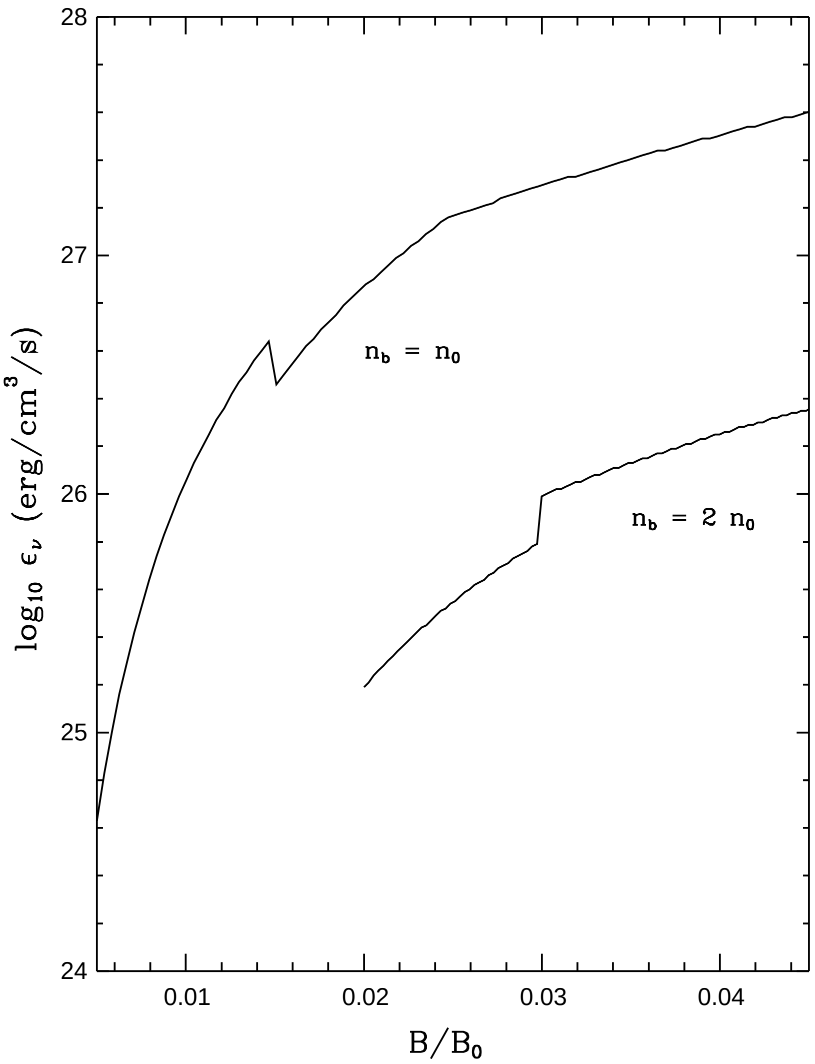

Figure 1: Neutrino emissivity of the direct URCA process as a function of the magnetic field intensity B (B in units of the proton critical field B_0), for two values of the baryon density and for a temperature $T = 10^9\ K$.

We now discuss the implications of the large emissivity produced by the URCA reactions in the presence of a strong magnetic field. Of course, in order to make detailed calculations, one needs to incorporate this emissivity into an elaborated neutron star cooling code. Aside from this, it is clear that other processes will also be modified in presence of the strong magnetic field (and maybe new processes will be possible). However, such considerations are out of the scope of this paper, as we discussed in the introduction. Here, we will only present some simple estimates.

As it is evident from fig. 1, the emissivity depends very strongly on the magnetic field intensity. One expects that an initial strong B will decay within a time t_B . Hence, even if the luminosity due to the URCA process is very high at the beginning, it will become very small for $t >> t_B$. According to [4], the characteristic decaying time for a superstrong magnetic field is $t_B \sim 100$ yr. In order to incorporate this effect, we have taken, as the time variation of B:

$$B(t) = B_{initial} e^{-t/t_B} \,. \tag{3.7}$$

Let us also assume that the thermal energy U_{th} of matter can be approximated by that of a degenerate neutron gas, then per unit volume one has

$$u_{th} = \frac{U_{th}}{V} = \frac{1}{2} A T^2 \,. \tag{3.8}$$

Since a neutron interacts with the external magnetic field only due to its anomalous magnetic moment, the thermal energy of a degenerate neutron gas will only slightly change under the action of the magnetic field. Neglecting this difference we assume $A = \left(\frac{\pi}{3}\right)^{2/3} n_b^{1/3} M_n^*$.

One has to solve the equation

$$\frac{du_{th}}{dt} = -\epsilon \,, \tag{3.9}$$

which can be rewritten as

$$\frac{dT}{dt} = -\frac{\epsilon}{(du_{th}/dT)} \,. \tag{3.10}$$

For our estimate, we will consider a fixed value of the density, with $n_b = n_0$. Then $u_{th} \sim 6.0 \times 10^{28} T_9^2 \ \mathrm{erg\,cm^{-3}}$. We have also chosen an initial value of the magnetic field $B_{initial} = 7 \times 10^{17} G$ and the temperature $T(0) = 10^9 K$. For the emissivity, we use the approximate formula eq. (3.6). Equation (3.10) can then be solved numerically. The result is plotted in fig. 2, where the vertical axes represents T_9 and the abscissa gives the time (in years). As seen from this figure, the temperature drops very fast at the beginning due to the URCA emission. However, as B goes to zero, the emission will decrease and the temperature stabilizes to a value which, in this case, is of the order of $10^8 K$. In fig. 3 we have plotted the URCA luminosity that arises from the above simplified model, with an assumed neutron star radius $R = 10^6 cm$. Here, one observes the large luminosity at early times and its rapid decrease due to the temperature drop. There is an ulterior secular decrease of the luminosity as a consequence of the magnetic field decay, so that L_{URCA} will become negligeable for $t >> t_B$. Standard emission processes will then dominate the neutron star cooling.

4. Conclusions

In this work we have considered the direct URCA process in degenerate nuclear matter under β-equilibrium in a strong magnetic field. These reactions are strongly suppressed by Pauli blocking in the field-free case, but we showed that, in a magnetic field $B \gtrsim k_F^2/(2e)$, direct URCA reactions are open for an arbitrary proton concentration, and will lead to a phase of fast neutron star cooling. As we mentioned in the introduction, the time duration of such a strong large-scale magnetic field decay via the mechanism

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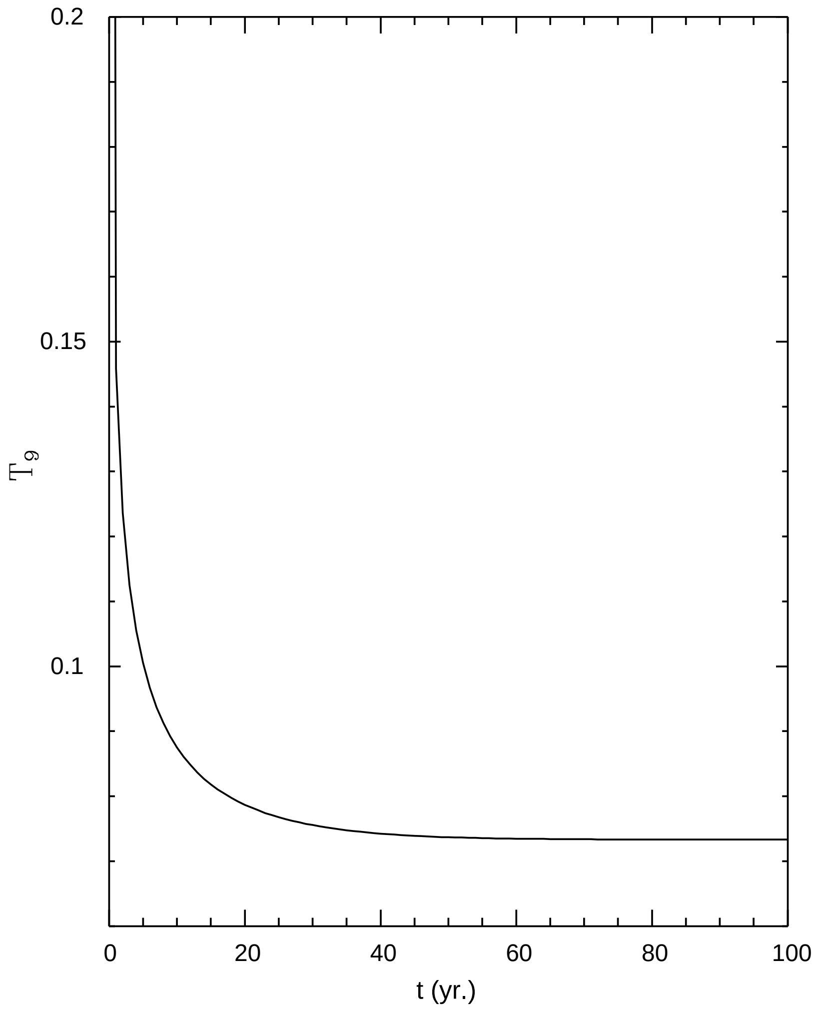

Figure 2: Temperature evolution, in units of $10^9 K$ for the simple model discussed on the text.

of ambipolar diffusion might be about a few decades or a hundred year [4]. If there are not other effects which disrupt the star during this time, our estimate shows that a superstrong magnetic field causes a very fast neutron star cooling. After 10 yr. or so, the neutron star temperature becomes one order of magnitude lower than in the field-free case, a result which is in discrepancy with observations of pulsar data.[2] One then can conclude that neutron stars with characteristic cooling times of the order $10^5 - 10^6$ years, as in the standard scenario, could not have an initial magnetic field comparable or larger than the critical value $B_c \gtrsim k_F^2/(2e)$, at least in the whole volume of the central core. On the other hand, if such stars exist, one has to search for anomalously cold neutron stars. Moreover, an ultra-high magnetic field as considered here might be able to produce a distortion on the shape of the neutron star and thus break the pulsar signal regularity. This fact seems also to disfavor such an enormous initial magnetic field.

[2] For a review on pulsar data, see for example [16].

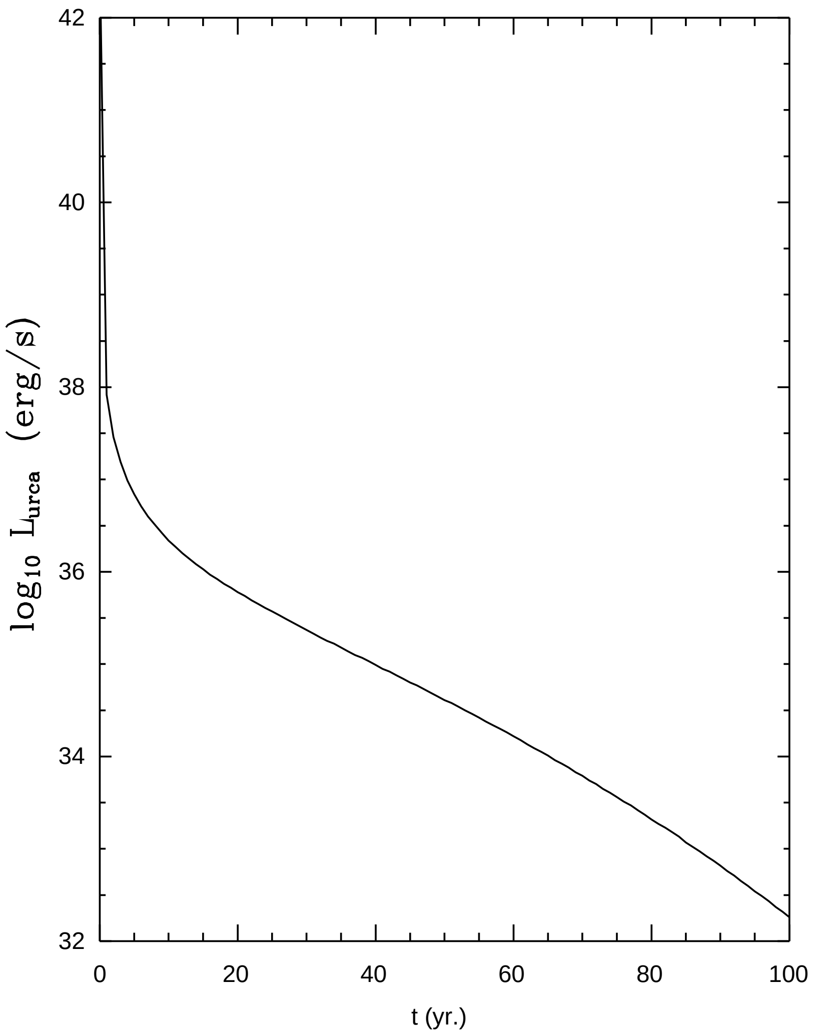

Figure 3: Luminosity of the URCA process (in erg/s) as a function of time, for the same model, in logarithmic scale.

In order to investigate the implications of this process, we have considered a very simple model. To simplify calculations we also restricted ourselves to the above values of the magnetic field. This approach makes possible to treat all protons and electrons as occupying only the ground Landau bands at zero temperature. However, as one can see from fig. 1, the direct URCA process can substantially contribute to neutron star cooling, even if the magnetic field is smaller than the value considered above. For the URCA processes to be effective one needs the electron momentum uncertainty $\Delta k_\perp \sim \sqrt{eB}$ to be of the order of the transversal momentum itself $k_\perp \sim \sqrt{2neB}$. This condition is still valid when electrons occupy a few Landau bands with $n \sim 1$. We also have considered matter densities not much larger than saturation densities. For these densities, nucleons can be treated as non-relativistic particles. More realistic calculations imply additional complications. If one intends to go to densities larger than the saturation density, then relativistic effects for nucleons should be taken into account and the many-level approximation should be used in order to calculate the energy loss via the direct URCA process. The standard processes should also be modified with taking into account a superstrong magnetic field. By making such a detailed simulation one could put more stringent bounds on the initial intensity of the magnetic field.

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